

Using the Statistical Probability of Coincident Peaks to Inform the Sizing of Transformers in Large Square Footage Residences

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Dominion[®]

Study Background

- **Data used are from an ongoing study of 103 mega-mansions by Dominion**
- **Original study started in the summer of 2007 and continues today**

Questions to be Addressed

- **Coincident Peaks Study**
 - What is the likelihood of all houses on a transformer reaching their peak at the same time?
 - What is the distribution of transformer peak load?
 - How do we determine transformer loading using information about peak load?
- **Duration of High Usage Study**
 - What is the behavior of transformer load around the peak load?
 - What are the factors that influence load level around the peak load?
- **Estimation of Usage by Square Footage Study**
 - In large homes, can usage pattern be estimated by the square footage of the home?
 - What other factors influence usage?

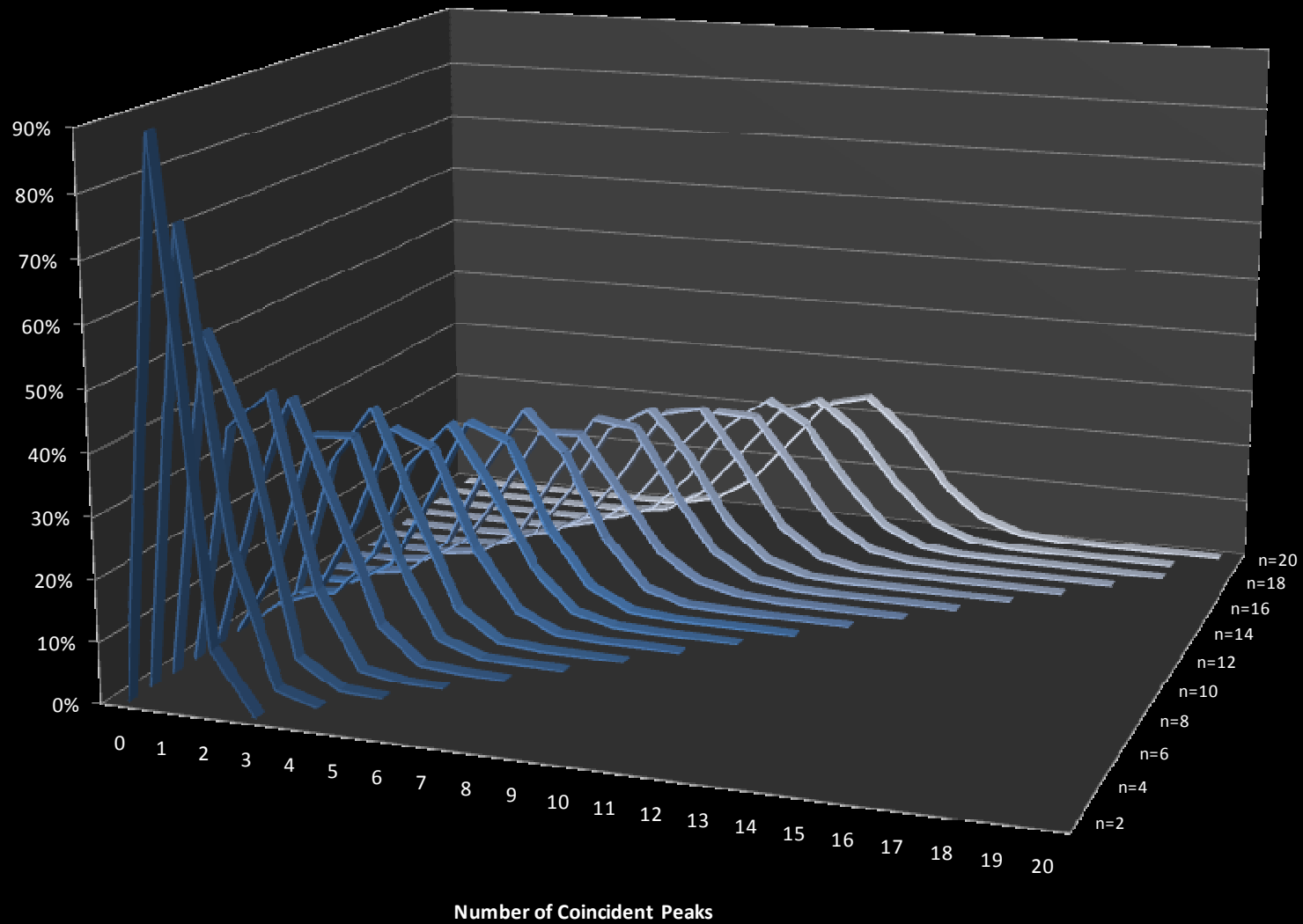
Coincident Peak and Transformer Loading Study

- **Original Data Set**
 - 23 houses within the same zip code in Northern Virginia
 - Houses similar with respect to square footage weather conditions
 - Half hourly data for June 13, 2007 through May 12, 2009
- **Methodology**
 - Randomly generated 100 samples for each of the possible sample sizes 2 through 20 to simulate the loading of transformers with 2 through 20 houses each
 - Daily sample peak, number of coincident individual peaks, and diversity factor for each of the 1900 simulated samples were determined
 - Daily sample peak was used to approximate the population distribution of sample peaks for each sample size
 - Statistical probability functions for coincident peaks were estimated
 - Using methodology proposed in the 2007 paper, “Statistical Properties of Diversity Factors for Probabilistic Loading of Distribution Transformers”, by Montgomery et al, probability of overload on a transformer was determined for each sample size

Coincident Peak Analysis

- **Probability distributions for number of coincident peaks are very right-skewed**
- **Probability of all houses peaking at the same time is very small for all sample sizes**
- **The maximum number of coincident peaks increases as the sample size increases, but at a lower rate than sample size**

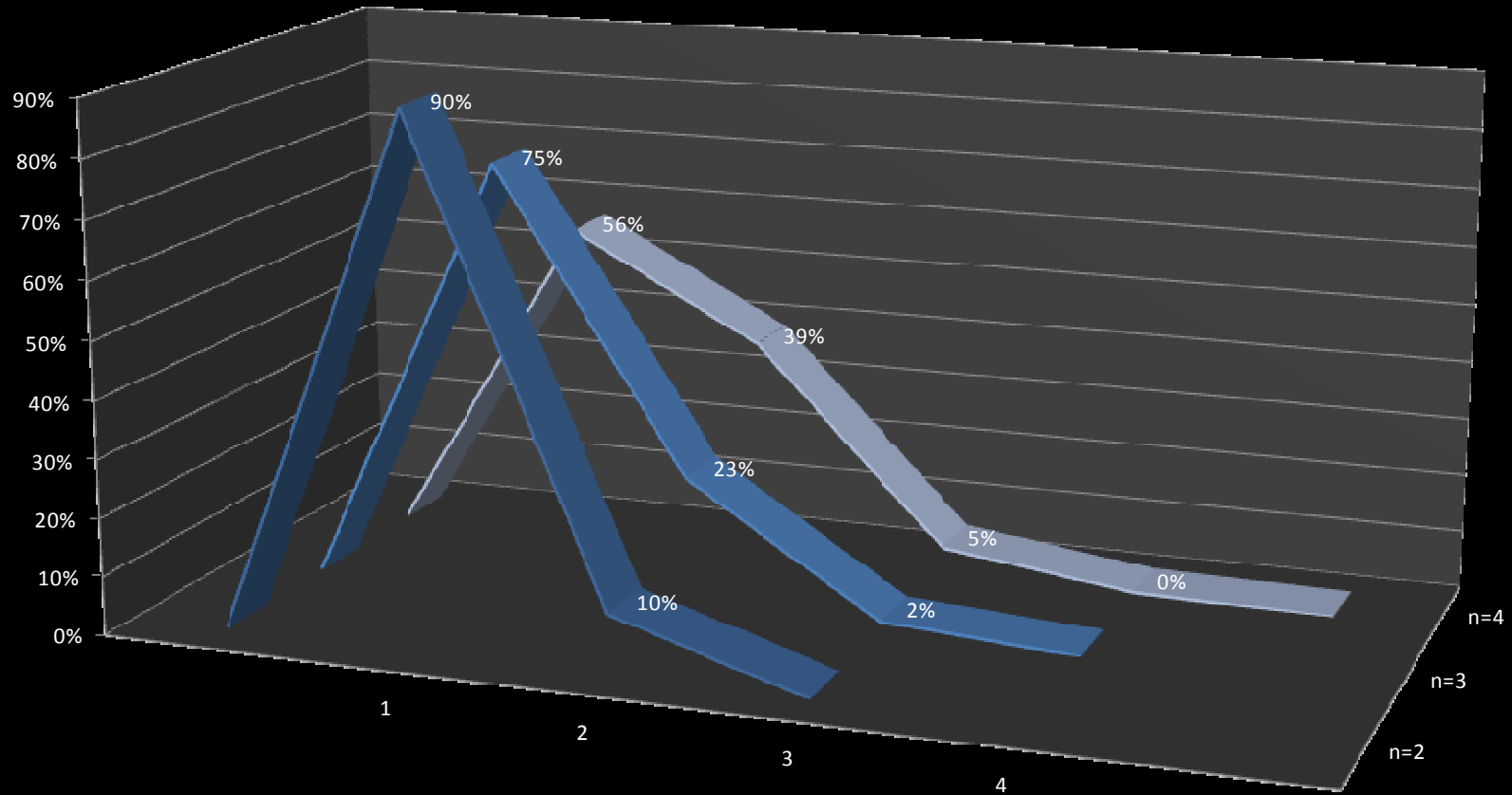
Probability of Coincident Peaks at Time of Sample Peak Up to 20 Homes per Transformer



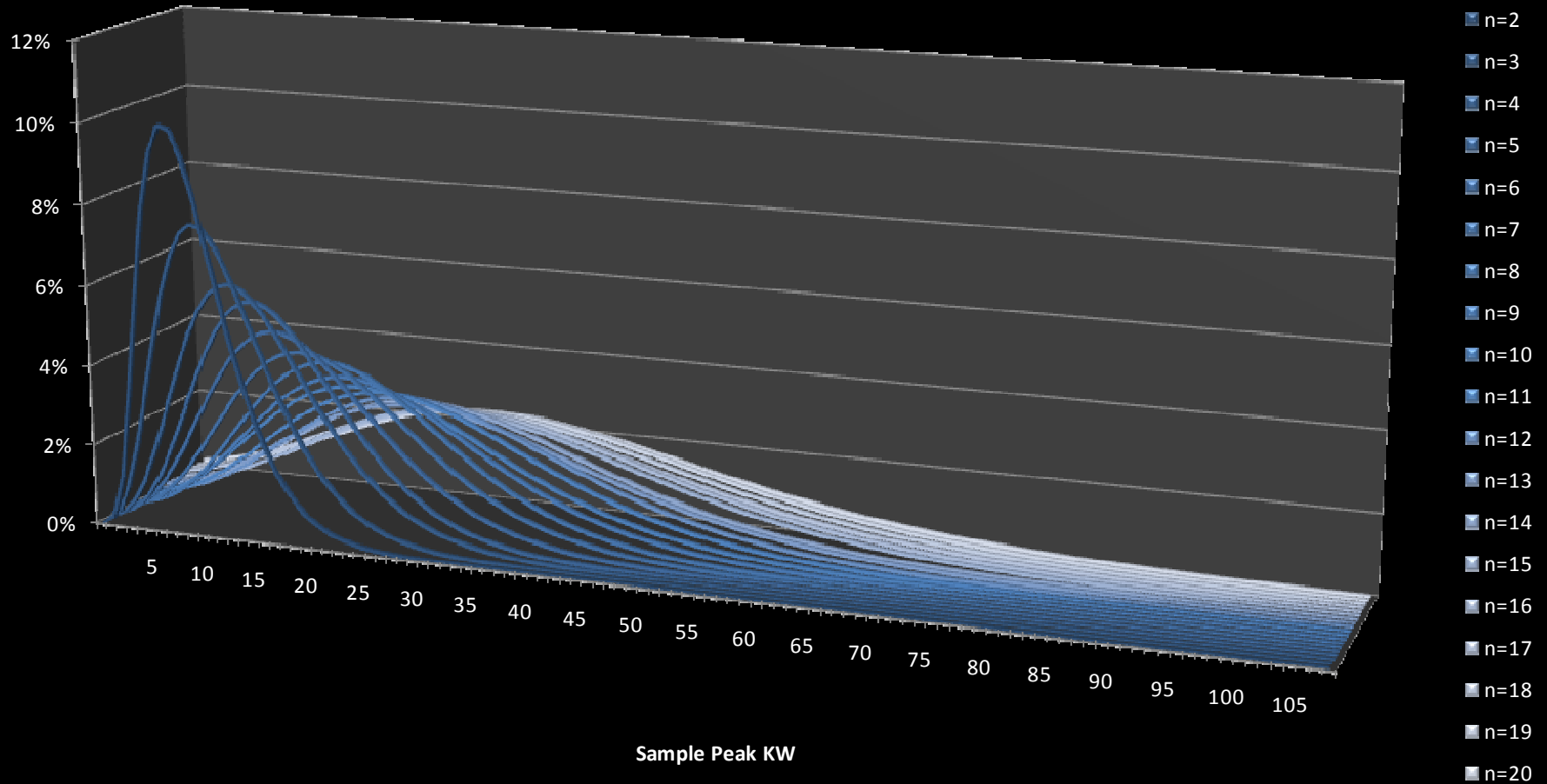
Probability of Coincident Peaks

Sample Size	Coincident Peaks																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	90%	10%																		
3	75%	23%	2%																	
4	56%	39%	5%	0%																
5	39%	45%	14%	1%	0%															
6	25%	43%	25%	6%	1%	0%														
7	12%	35%	35%	15%	3%	0%	0%													
8	5%	29%	38%	22%	5%	1%	0%	0%												
9	3%	16%	33%	30%	14%	4%	1%	0%	0%											
10	1%	8%	25%	32%	22%	9%	2%	0%	0%	0%										
11	0%	4%	17%	31%	28%	14%	4%	1%	0%	0%	0%									
12	0%	1%	7%	21%	32%	25%	11%	2%	0%	0%	0%	0%								
13	0%	1%	5%	15%	26%	26%	17%	7%	2%	1%	0%	0%	0%							
14	0%	0%	1%	6%	17%	27%	26%	15%	6%	1%	0%	0%	0%	0%						
15	0%	0%	0%	3%	12%	24%	28%	20%	9%	3%	1%	0%	0%	0%	0%					
16	0%	0%	0%	1%	5%	14%	25%	27%	18%	7%	2%	0%	0%	0%	0%	0%				
17	0%	0%	0%	0%	2%	8%	17%	24%	24%	15%	6%	2%	0%	0%	0%	0%	0%			
18	0%	0%	0%	0%	1%	3%	10%	20%	26%	21%	12%	5%	1%	0%	0%	0%	0%	0%		
19	0%	0%	0%	0%	0%	1%	5%	13%	21%	24%	19%	10%	4%	1%	0%	0%	0%	0%	0%	0%
20	0%	0%	0%	0%	0%	0%	2%	6%	14%	22%	24%	18%	9%	3%	1%	0%	0%	0%	0%	0%

Probability of Coincident Peaks at Time of Sample Peak Four or Fewer Houses per Transformer



Distributions of Sample Peaks by Sample Size



Determining probability of Transformer Overload Using Distributions of Transformer Rating and Sample Peaks

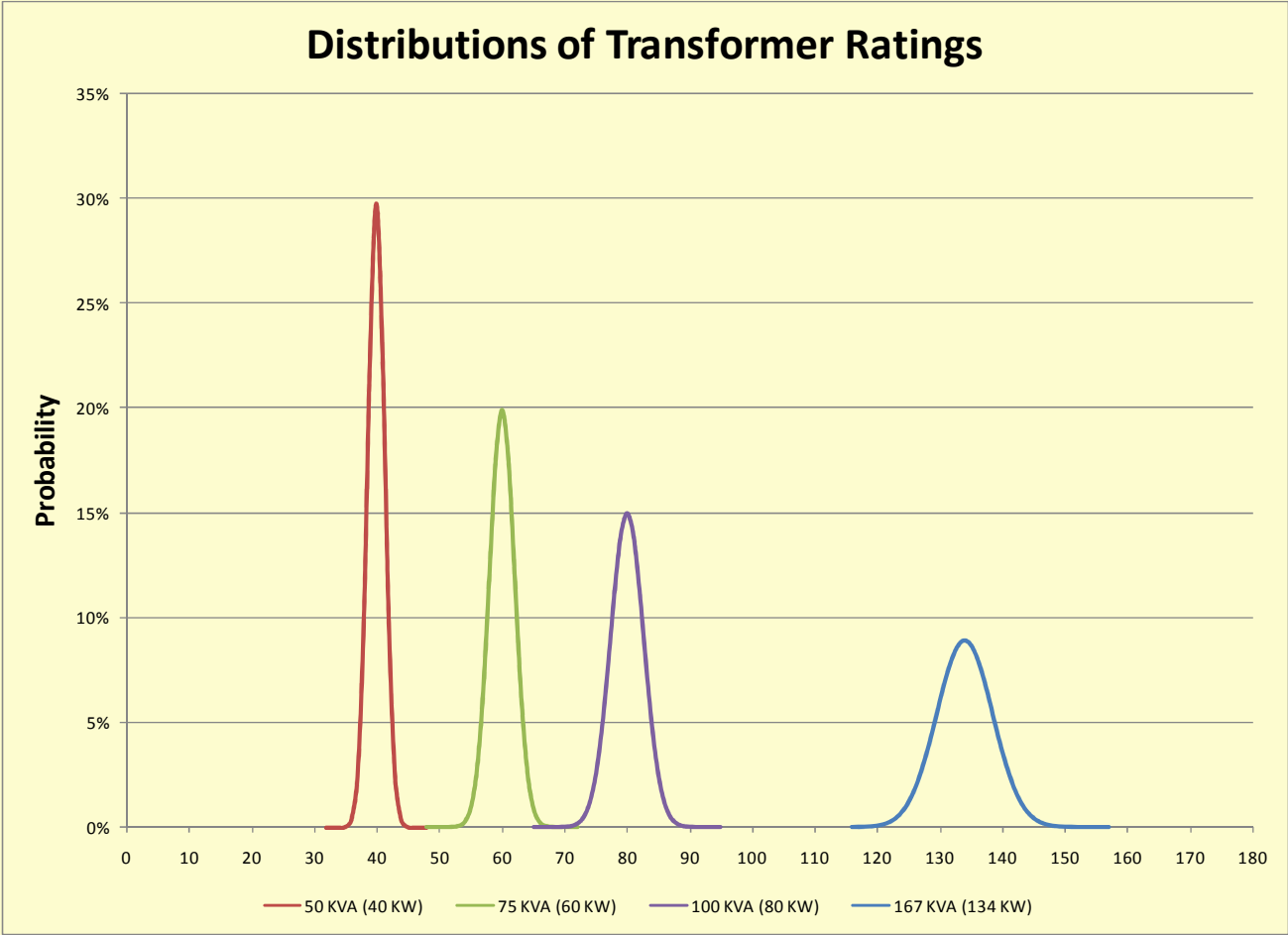
- **Distributions of Transformer Rating**
 - From the paper by Montgomery, et al, the probability distribution of 50 KVA transformer is normal with mean = 50 KVA and standard deviation = 1.67 KVA
 - We will assume distributions for 75 KVA, 100 KVA and 167 KVA transformers have similar coefficients of variation to determine means and standard deviations
 - Since customer data is in KW, we will convert KVA to KW
- **Distributions of Sample Peaks**
 - Also from the Montgomery paper, the distribution of sample peaks for a given sample size should approximate a Gamma distribution
 - Parameters for the appropriate Gamma distribution can be determined using Proc Univariate in SAS

Conversion of KVA to KW

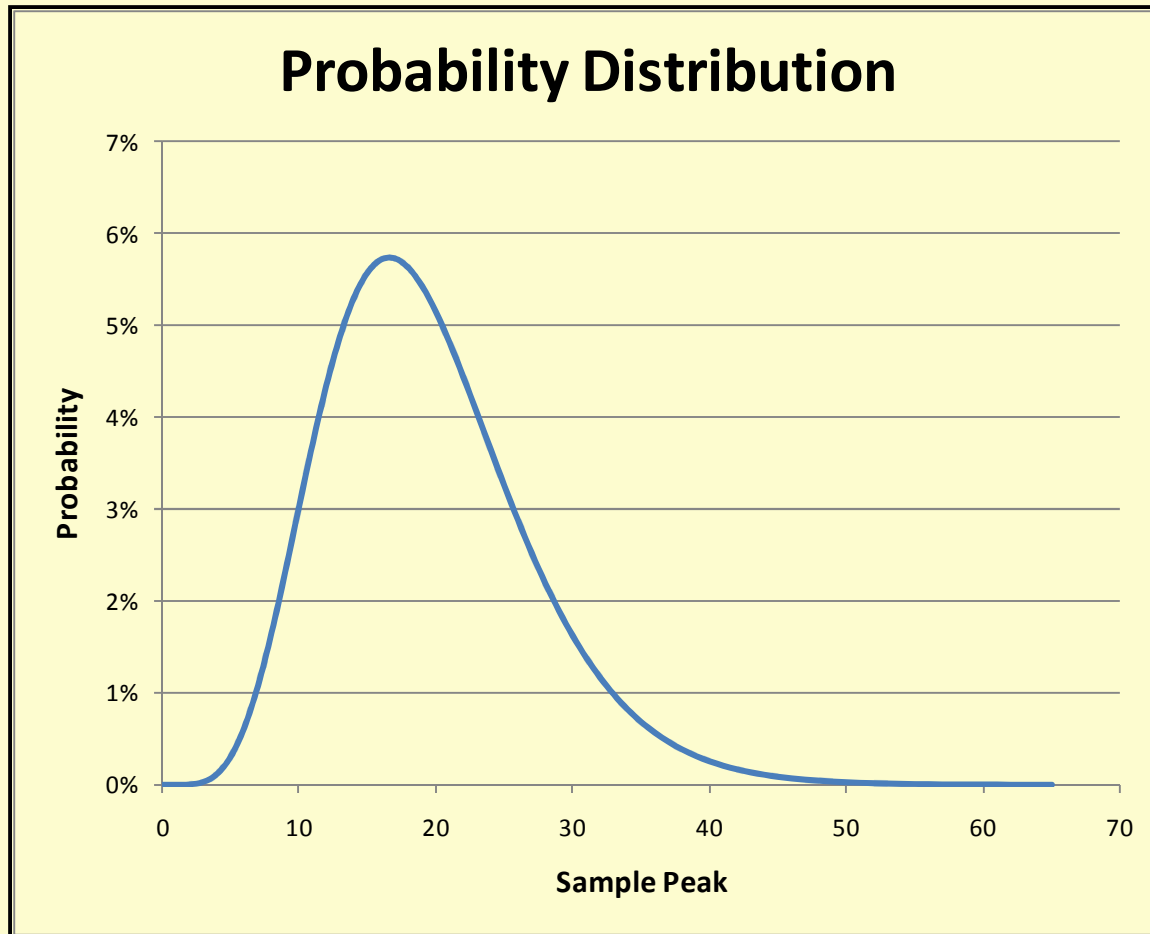
	Transformer Rating (KVA)			
	50	75	100	167
Average	50	75	100	167
Standard Deviation	1.67	2.5	3.33	5.57

	Transformer Rating (KW)			
	40	60	80	134
Average	40	60	80	134
Standard Deviation	1.34	2.00	2.66	4.46

Transformer Rating Distributions

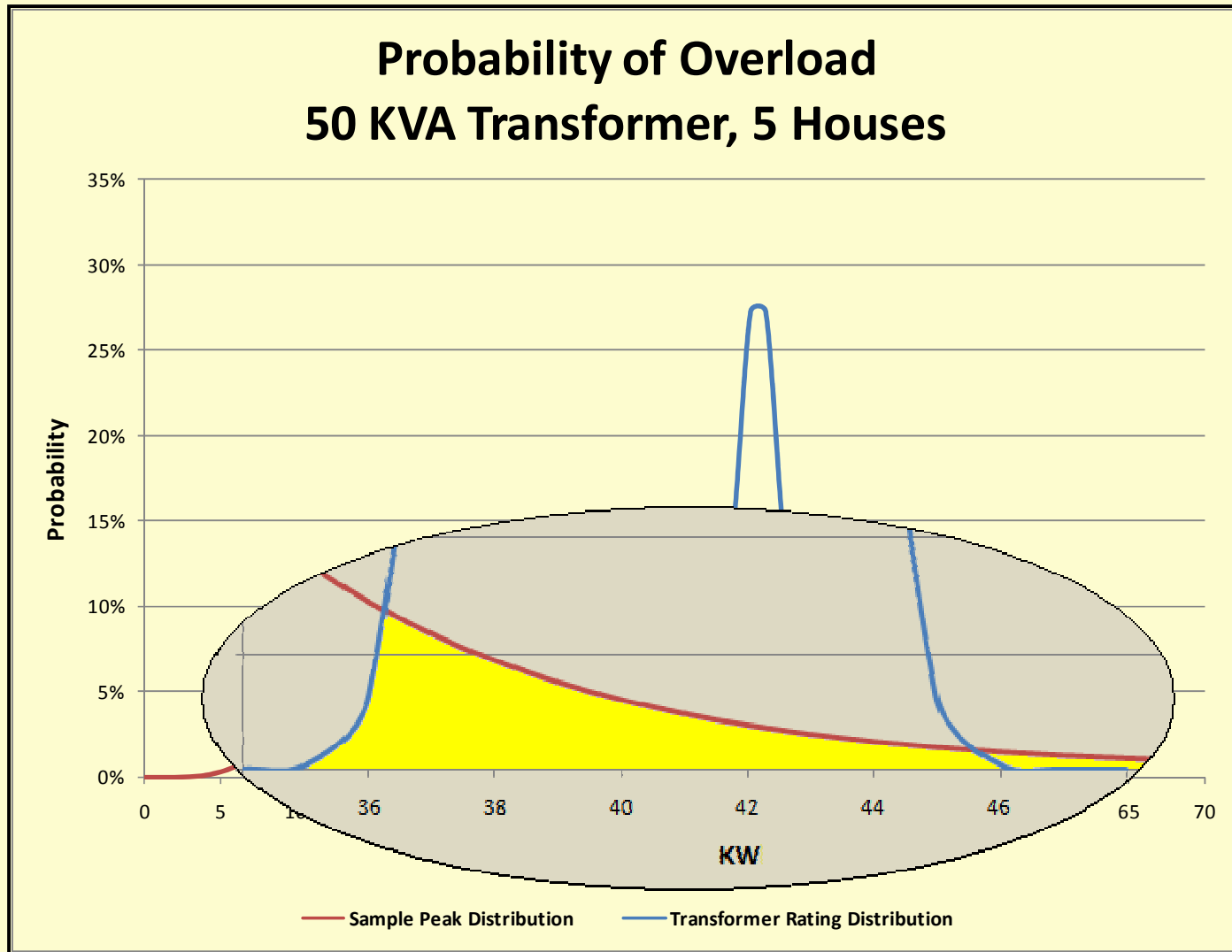


Distribution of Sample Peaks, $n=5$ Using the Gamma Approximation

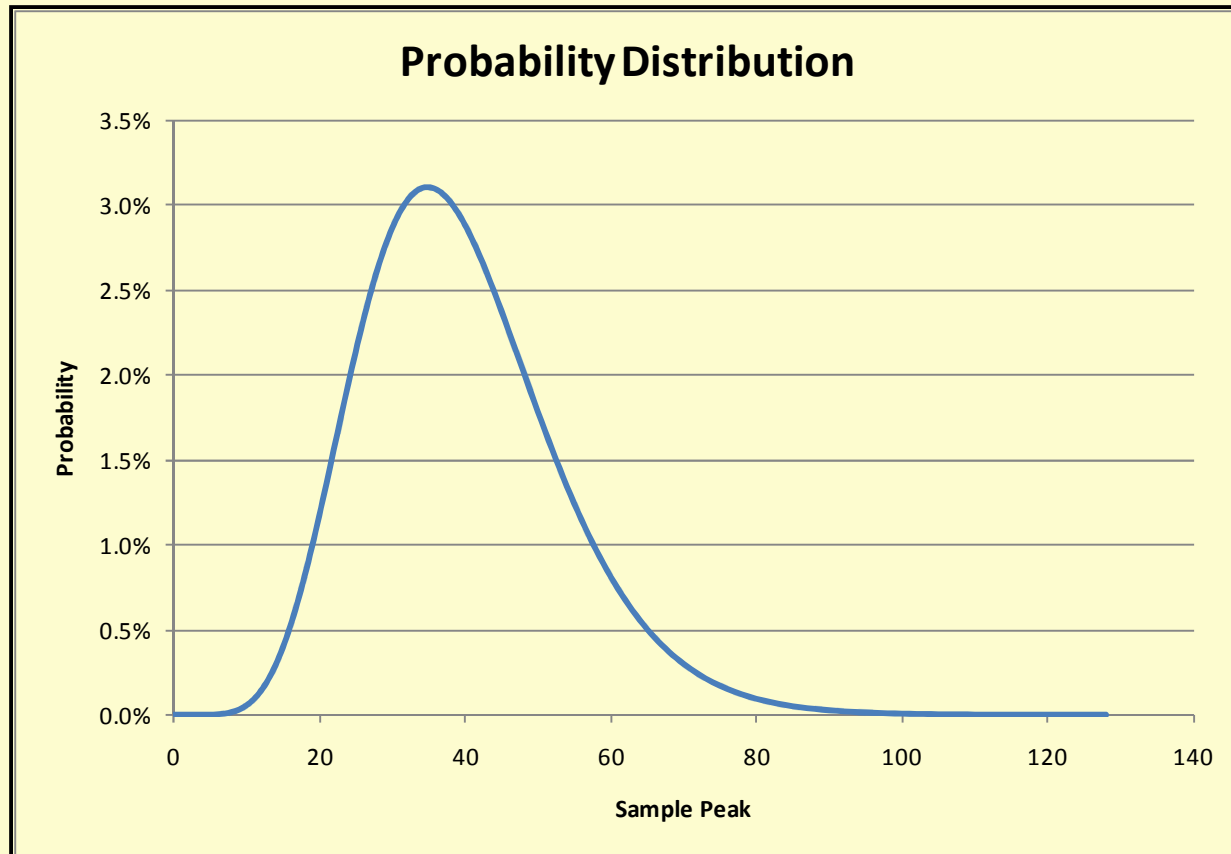


Individual Daily Peak	
Mean	5.17
Standard Deviation	2.81
Sample Daily Peak	
Mean	19.43
Standard Deviation	7.41

Probability of Overload for n=5 Homes on 50 KVA Transformer is 1.2%

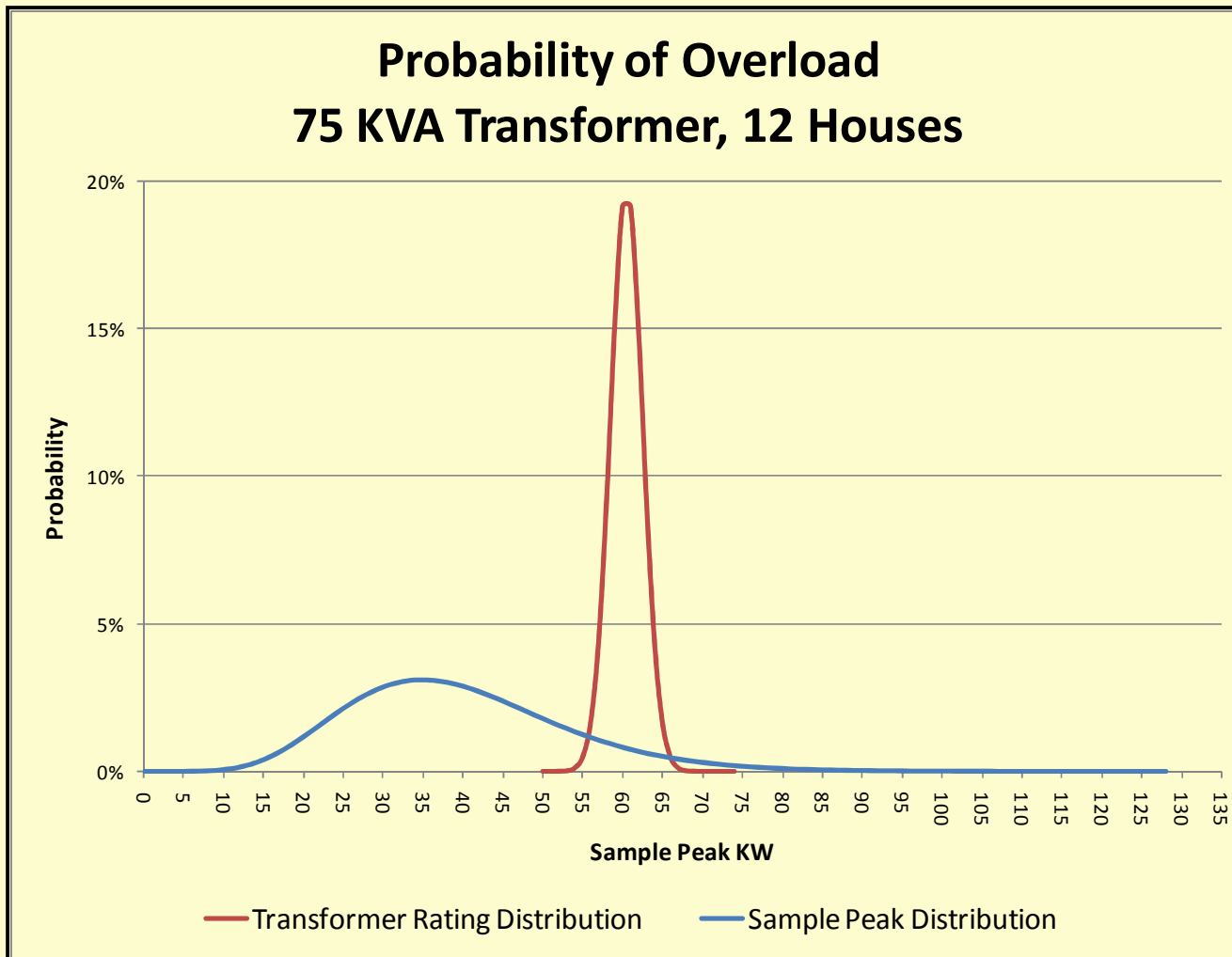


Distribution of Sample Peaks, n=12 Using the Gamma Approximation

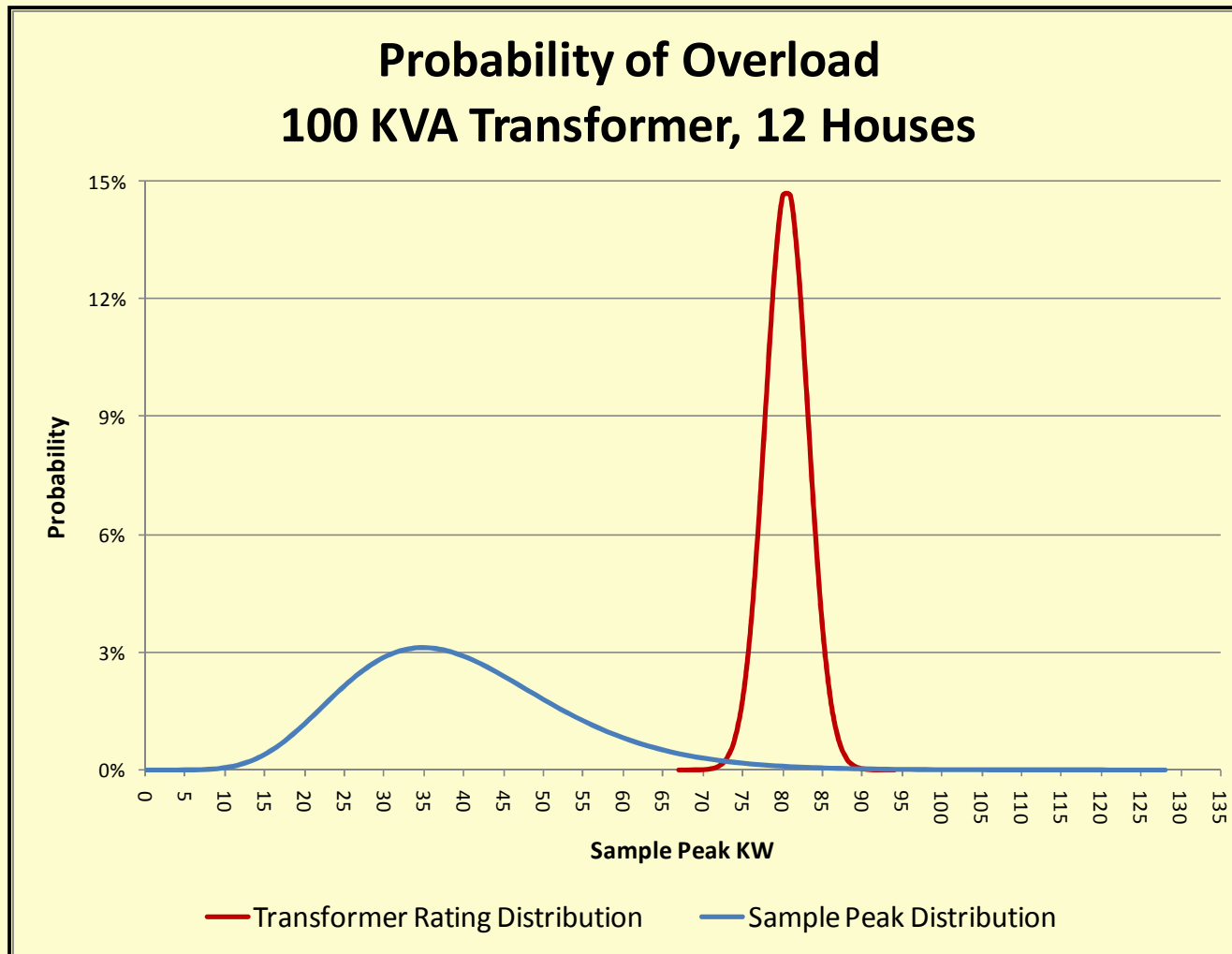


Individual Daily Peak	
Mean	5.17
Standard Deviation	2.81
Sample Daily Peak	
Mean	39.51
Standard Deviation	13.53

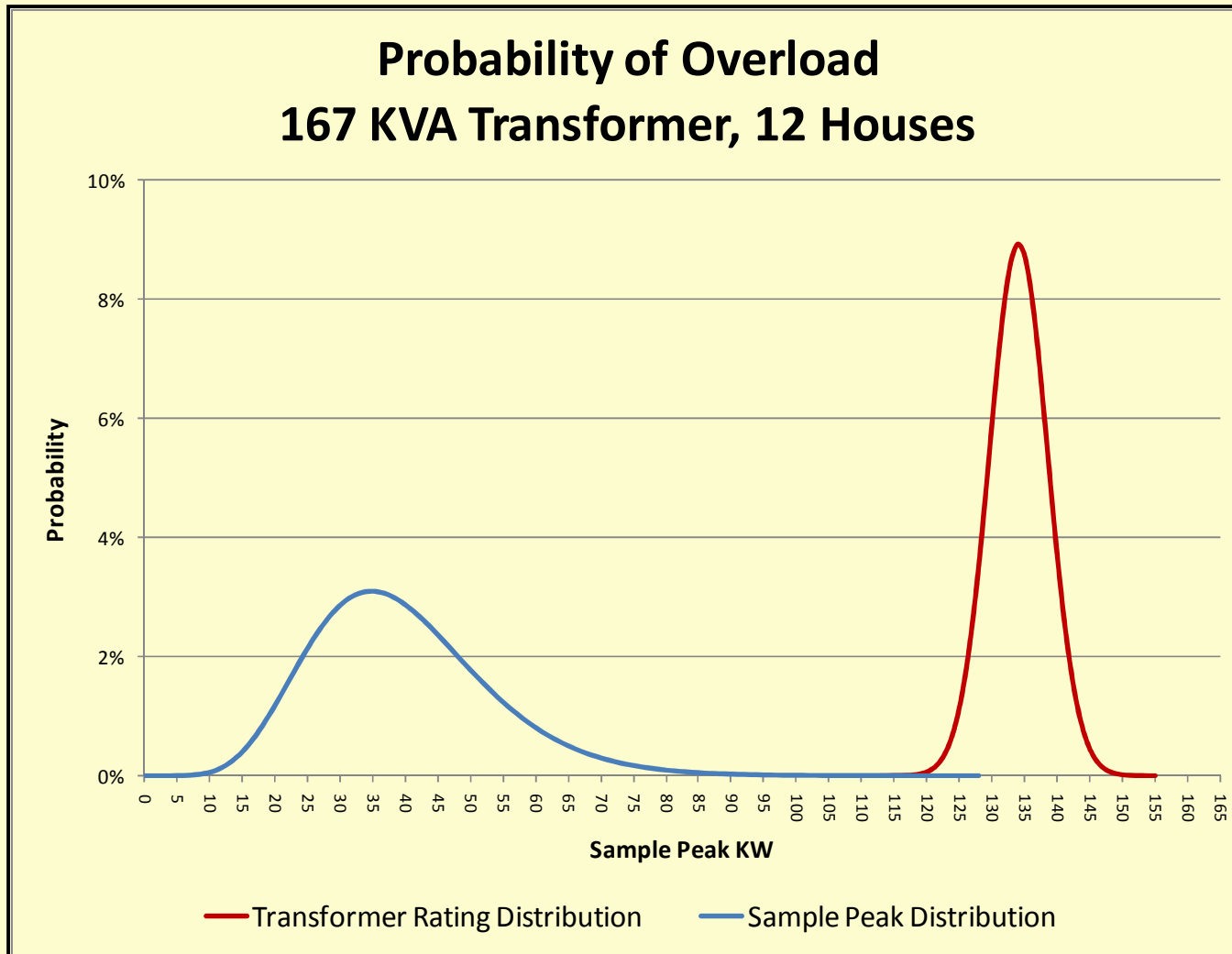
Probability of Overload for n=12 Homes on 75 KVA Transformer is 7.9%



Probability of Overload for n=12 Homes on 100 KVA Transformer is 0.77%



Probability of Overload for n=12 Homes on 167 KVA Transformer is essentially 0%



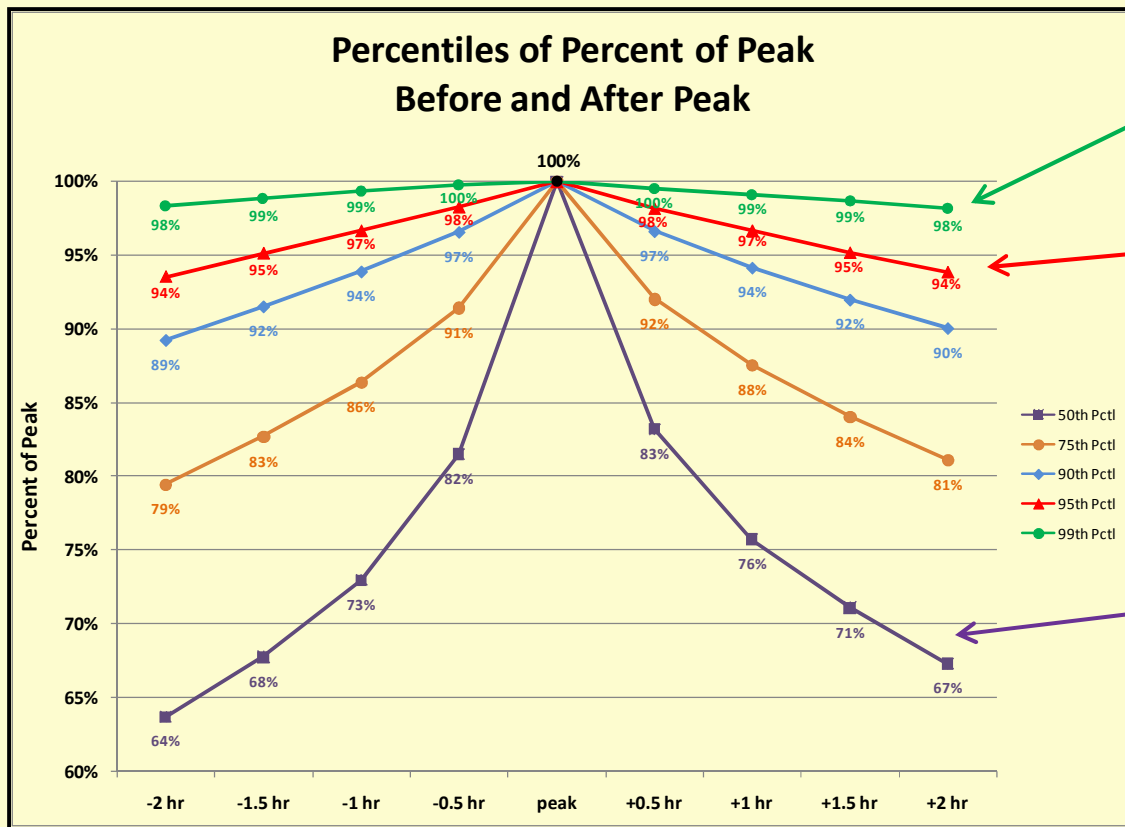
Transformer Loading: Probability of Overload $\leq 0.5\%$

Number of Houses on Transformer	Transformer Rating			
	50 KVA (40 KW)	75 KVA (60 KW)	100 KVA (80 KW)	167 KVA (134 KW)
2	0.0%	0.0%	0.0%	0.0%
3	0.0%	0.0%	0.0%	0.0%
4	0.5%	0.0%	0.0%	0.0%
5	1.2%	0.0%	0.0%	0.0%
6	3.8%	0.1%	0.0%	0.0%
7	8.3%	0.3%	0.0%	0.0%
8	13.8%	0.7%	0.0%	0.0%
9	21.0%	1.8%	0.1%	0.0%
10	27.0%	3.1%	0.2%	0.0%
11	35.9%	5.2%	0.4%	0.0%
12	44.1%	7.9%	0.8%	0.0%
13	48.9%	11.3%	1.5%	0.0%
14	52.4%	12.2%	1.6%	0.0%
15	62.2%	18.9%	3.3%	0.0%
16	66.0%	21.8%	4.2%	0.0%
17	66.9%	24.0%	5.3%	0.0%
18	71.2%	27.5%	6.4%	0.0%
19	74.9%	31.1%	7.8%	0.0%
20	76.2%	33.5%	9.1%	0.1%

Duration of High Usage

- **Data for study is half hourly load information for the 103 houses in the large square footage house study**
- **Four half hour intervals around the daily peak for each house were analyzed.**
- **Data were expressed as percent of the peak load**

Load Ramps Up to Peak and Then Declines in the 2 Hour Interval Around Peak



Less than 1% of the time, the load is 98% or more of peak in the 2 hours surrounding the peak

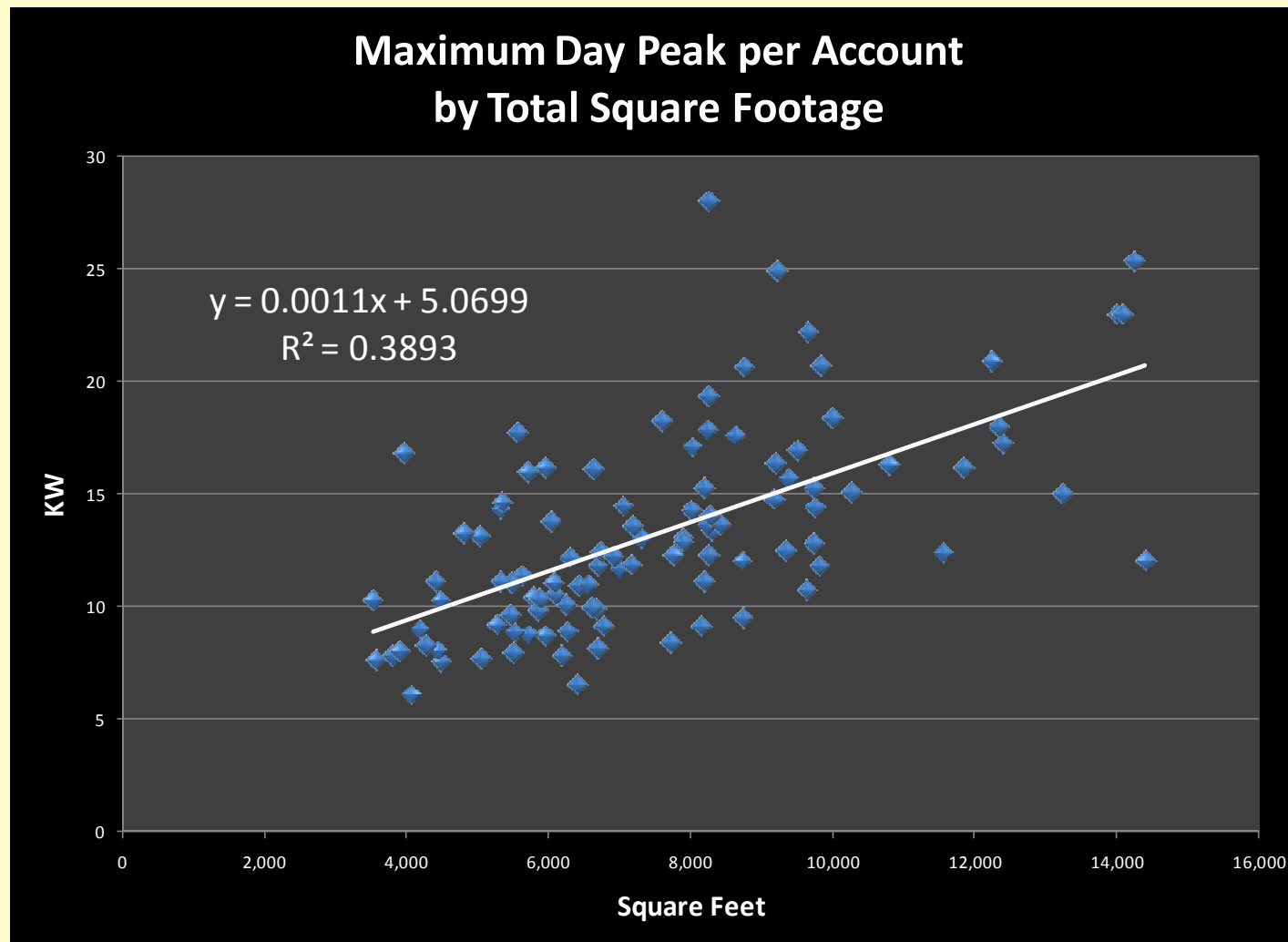
5% of the time, the load stays in the 94%-98% of peak range in the 2 hours surrounding the peak

At least 50% of the time, the load drops off from 83% to 67% of peak in the 2 hours surrounding the peak

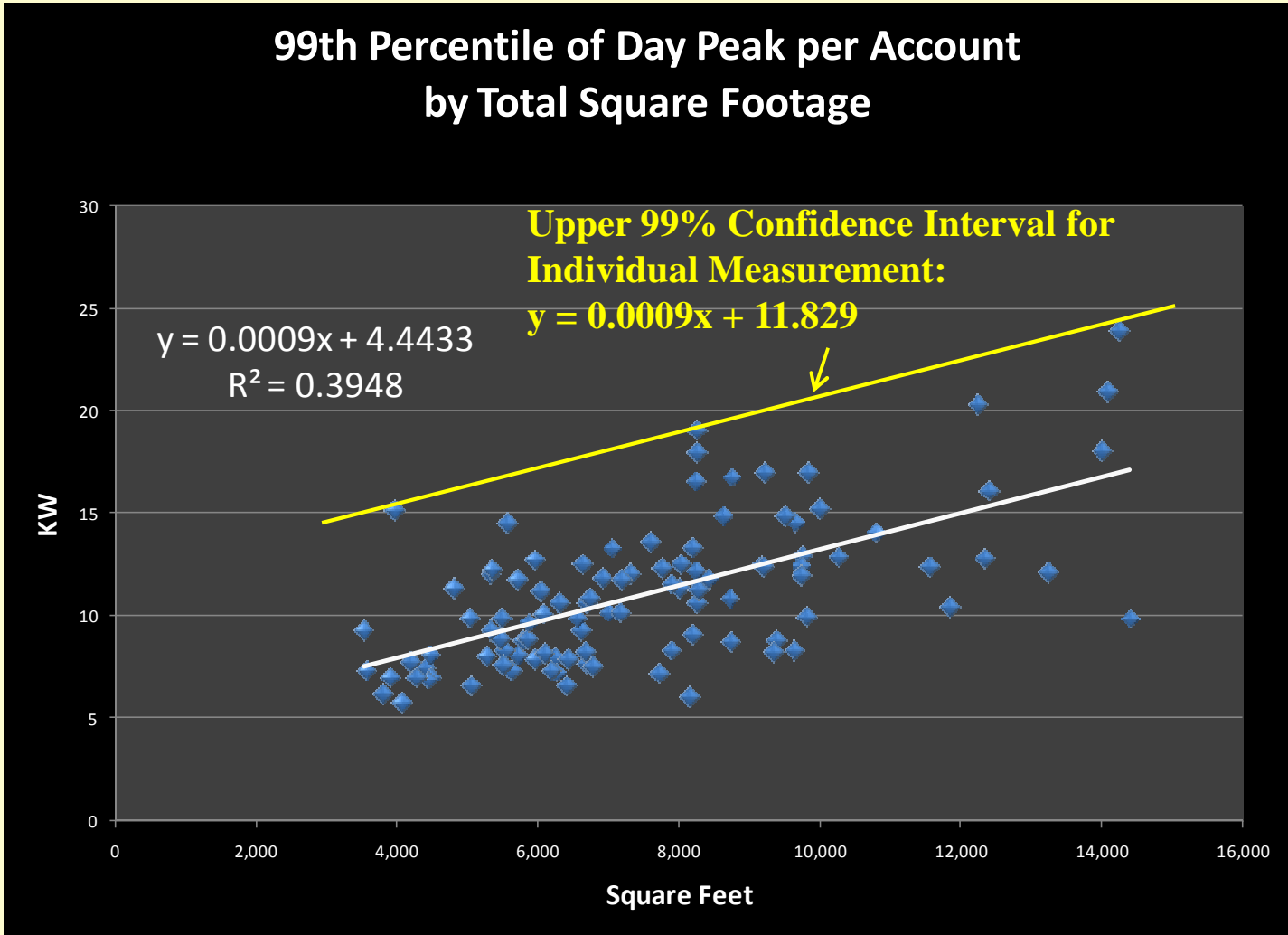
Estimating Electrical Usage as a Function of Square Footage

- **Data for study is half hourly load information for the 103 houses in the large square footage house study for dates June 01,2007 through May 12,2009**
- **Descriptive statistics were calculated by house for the data including:**
 - **Maximum of half hourly load for each house across the time period**
 - **99th percentile of half hourly load for each house across the time period**
- **Both the maximum load and the 99th percentile show a fairly strong linear relationship with the total square footage of the house**
- **Other variables (number of bedrooms, number of bath rooms, stories, etc) were explored as possible predictors, but none were found to significantly predict either the maximum or 99th percentile**

Account day peak shows a linear pattern with one or two extreme values



99th Percentile has a more compact pattern around the linear regression line.



Summary

- **Coincident Peaks Study**
 - Using the probability of coincident peaks for houses on a single transformer and the distribution of transformer rating, the probability of transformer overload can be determined statistically
 - Probability of transformer overload under differing loading scenarios can be used to aid in transformer loading
- **Duration of High Usage Study**
 - Duration of peak load around the transformer peak can be described as a percentage of the peak load
 - Duration of peak load can be used to further add to information about transformer loading
- **Estimation of Usage by Square Footage Study**
 - In large homes, usage pattern, in particular maximum load, can be estimated by the square footage of the home
 - Other demographic factors were not shown to influence usage*

Note: Data used for this study had very limited demographic information.

Combining Usage by Square Footage with Probability of Overload

Number of Houses	50 KVA					100 KVA					167 KVA				
	7000	8000	9000	10000	11000	7000	8000	9000	10000	11000	7000	8000	9000	10000	11000
2	0.00%	0.00%	0.00%	0.01%	0.03%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
3	0.00%	0.00%	0.02%	0.06%	0.14%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
4	0.00%	0.01%	0.08%	0.24%	0.45%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
5	0.00%	0.04%	0.26%	0.66%	1.09%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
6	0.01%	0.14%	0.66%	1.47%	2.18%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
7	0.03%	0.38%	1.40%	2.75%	3.79%	0.00%	0.00%	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%
8	0.08%	0.82%	2.56%	4.56%	5.92%	0.00%	0.00%	0.00%	0.01%	0.02%	0.00%	0.00%	0.00%	0.00%	0.00%
9	0.19%	1.55%	4.21%	6.91%	8.54%	0.00%	0.00%	0.00%	0.02%	0.04%	0.00%	0.00%	0.00%	0.00%	0.00%
10	0.40%	2.62%	6.35%	9.73%	11.57%	0.00%	0.00%	0.01%	0.04%	0.09%	0.00%	0.00%	0.00%	0.00%	0.00%
11	0.74%	4.08%	8.95%	12.96%	14.92%	0.00%	0.00%	0.02%	0.08%	0.17%	0.00%	0.00%	0.00%	0.00%	0.00%
12	1.26%	5.91%	11.93%	16.48%	18.48%	0.00%	0.00%	0.04%	0.14%	0.29%	0.00%	0.00%	0.00%	0.00%	0.00%
13	1.98%	8.11%	15.23%	20.20%	22.17%	0.00%	0.01%	0.08%	0.25%	0.46%	0.00%	0.00%	0.00%	0.00%	0.00%
14	2.93%	10.63%	18.73%	24.03%	25.89%	0.00%	0.02%	0.13%	0.40%	0.70%	0.00%	0.00%	0.00%	0.00%	0.00%
15	4.11%	13.39%	22.36%	27.86%	29.58%	0.00%	0.03%	0.22%	0.60%	1.00%	0.00%	0.00%	0.00%	0.00%	0.00%
16	5.50%	16.34%	26.03%	31.65%	33.19%	0.00%	0.06%	0.34%	0.86%	1.38%	0.00%	0.00%	0.00%	0.00%	0.01%
17	7.09%	19.41%	29.67%	35.32%	36.66%	0.00%	0.09%	0.49%	1.18%	1.83%	0.00%	0.00%	0.00%	0.00%	0.01%
18	8.86%	22.54%	33.23%	38.84%	39.97%	0.01%	0.14%	0.70%	1.57%	2.34%	0.00%	0.00%	0.00%	0.01%	0.02%
19	10.76%	25.66%	36.66%	42.17%	43.09%	0.01%	0.21%	0.94%	2.02%	2.92%	0.00%	0.00%	0.00%	0.01%	0.02%
20	12.76%	28.74%	39.93%	45.31%	46.02%	0.02%	0.30%	1.23%	2.53%	3.55%	0.00%	0.00%	0.00%	0.01%	0.04%

APPENDIX

Some of Homes in Transformer Loading Study



Non-Subdivision Homes Excluded from Analysis

